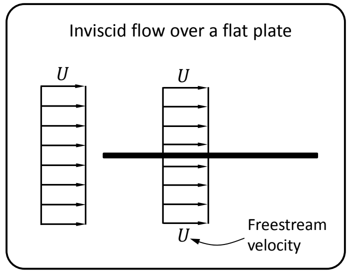
ME 306 Fluid Mechanics II

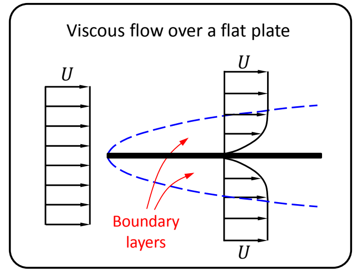
Part 2

Viscous Flow over Immersed Bodies

These presentations are prepared by
Dr. Cüneyt Sert
Department of Mechanical Engineering
Middle East Technical University
Ankara, Turkey
csert@metu.edu.tr

Boundary Layer (BL) Concept





- In 1904 Ludwig Prandtl proposed the concept of boundary layer (BL).
- BLs are the regions where viscous effects are dominant. They develop over solid surfaces, in the wake regions of bodies, mixing layers and jet flows.
- Outside the BLs velocity gradients are low and the flow can be treated as inviscid (frictionless).

BL and Reynolds Number (Re)

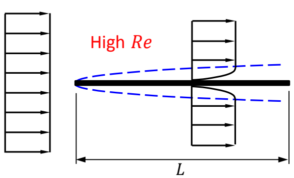
$$Re_L = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{(\text{mass}) \times (\text{acc.})}{(\text{visc.}) \times (\text{vel. grad.}) \times (\text{area})} \frac{\rho L^2 U^2}{\mu U L} = \frac{\rho U L}{\mu} = \frac{UL}{\nu}$$

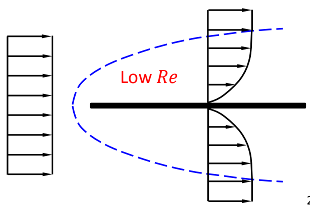
High Re flow

- Inertia forces are dominant over a large portion of the flow field.
- BL is thin.

Low Re flow

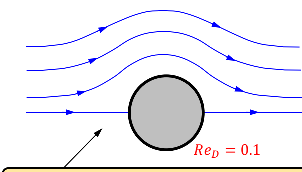
- Viscous forces can be felt over a broader region.
- BL is thick.



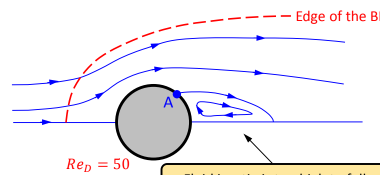


BL and Reynolds Number (cont'd)

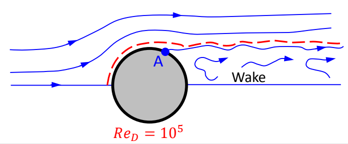
- For blunt (not streamlined) bodies, like a cylinder, formation of wake region and BL separation depend on Re.



- Viscous effects are important over a very large domain. It is not proper to talk about a BL.
- Streamlines are similar to those of potential flow.



- Fluid inertia is too high to follow the contour of the cylinder.
- Separation occurs at point A. Circulation bubble forms behind.



- BL is very thin and attached at the front.
- BL separates at the rear side.
- There is a large wake region behind.

BL and Reynolds Number (cont'd)

Exercise : What is the Reynolds number for an inviscid flow? How does such a flow over a flat plate look like?

Exercise : Estimate the Reynolds numbers for the following flows

a) Car traveling at a speed of 100 km/h.

b) An airliner travelling at a typical cruising speed and altitude.

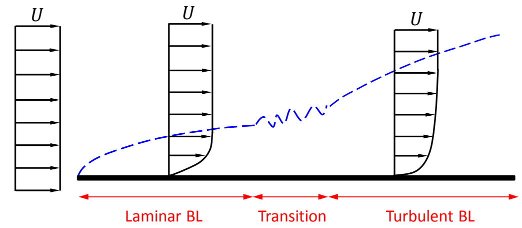
c) Golf ball hit hard by a professional golfer.

d) Fuel injected into an internal combustion engine's cylinder.

e) Blood flowing in human aorta and air flowing in human trachea.

Exercise : Think about problems of engineering importance where the Reynolds number is very low (in the order of unity).

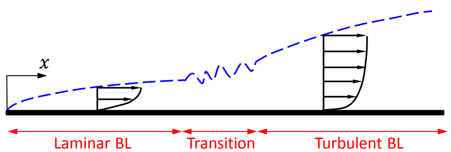
BL over a Flat Plate



Warning
This drawing is out of scale. Typically BLs are very thin.

- As the BL grows, small disturbances in the flow also grow.
- At some point they can no longer be damped by the viscous action and transition to turbulence takes place.
- Growth rate of turbulent BL is larger than that of laminar BL.
- In a turbulent BL, due to enhanced mixing, fluid with high momentum comes closer to the flat plate. Velocity gradient and therefore shear stress at the plate becomes higher compared to those of laminar BL.

BL over a Flat Plate (cont'd)

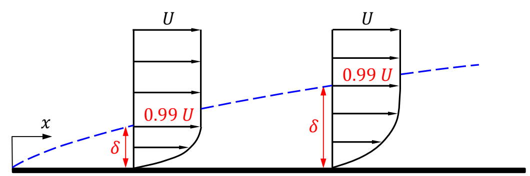


- Transition to turbulence can be checked with the local Reynolds number

$$Re_x = \frac{Ux}{\nu}$$

- Critical Re_x for transition to turbulence on a flat plate is accepted to be 5×10^5.
- Factors that affect turbulence transition are
  - initial intensity of upstream turbulence
  - roughness of the plate
  - condition (sharpness, smoothness) of the leading edge
  - flow unsteadiness
  - any other disturbances such as vibrations, acoustic noise, heat transfer, etc.

BL Thickness (δ)

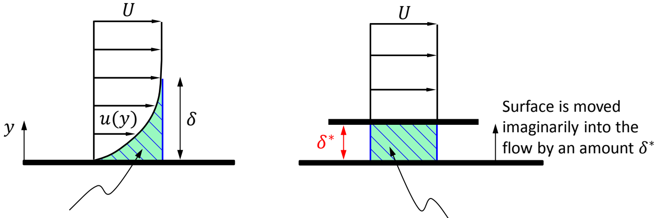


- BL thickness (δ) is the distance from the plate where the velocity reaches to 99 % of the free stream velocity U.
- δ is a function of x. It increases with x.

Exercise : By considering a proper control volume inside the BL, show that the dashed line denoting the edge of the BL is NOT a streamline. Draw streamlines on the figure given above.

BL Displacement Thickness (δ^*)

- δ^* is the distance a solid surface should be imaginarily moved into an inviscid flow so that the created mass flux loss is equal to that caused inside the BL of the actual viscous flow.



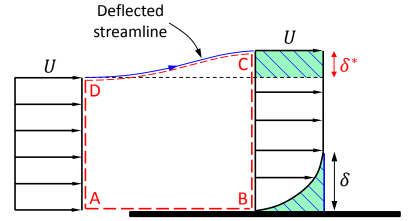
$$\dot{m}_{loss} = \int_{y=0}^{\delta} \rho(U-u)\, w\, dy \qquad \dot{m}_{loss} = \rho\, U\, \delta^* w$$

Constant width of the surface

$$\delta^* = \int_{y=0}^{\delta} \left(1 - \frac{u}{U}\right) dy$$

BL Displacement Thickness (cont'd)

- Alternative physical interpretation of δ^* is the shift of the flow (deflection of the streamlines) away from the surface due to the presence of the BL.



Consider the mass conservation for the control volume ABCD.

Shaded areas should be equal, which gives the equation for δ^*.

$$\dot{m}_{AB} = 0$$

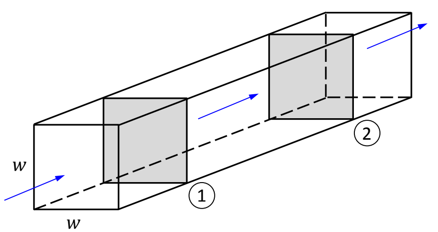
$$\dot{m}_{CD} = 0$$

$$|\dot{m}_{AD}| = |\dot{m}_{BC}|$$

BL Displacement Thickness (cont'd)

Exercise : A laboratory wind tunnel has a 0.5 x 0.5 m² square test section. BL velocity profiles are measured at two axial locations and displacement thicknesses are determined. At section 1, where the freestream speed is 30 m/s, the displacement thickness is 1.5 mm. At section 2, located downstream from section 1, displacement thicknesss is 2.5 mm.

Calculate the change in static pressure between sections 1 and 2.



$w = 0.5$ m

$U_1 = 30$ m/s

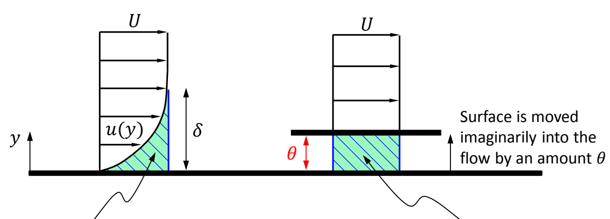
$\delta_1^* = 1.5$ mm

$\delta_2^* = 2.5$ mm

$p_1 - p_2 = ?$

BL Momentum Thickness (θ)

- θ is the distance a solid surface should be imaginarily moved into inviscid flow so that the created momentum flux loss is equal to that caused inside the BL of the actual viscous flow.



$$\text{momentum loss} = \int_{y=0}^{\delta} \rho u(U-u)\, w dy \qquad \text{momentum loss} = \rho U^2 \theta w$$

Constant width of the surface

$$\theta = \int_{y=0}^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Prandtl's BL Equations

- Consider steady, incompressible flow over a surface.
- Continuity and Navier-Stokes equations are

Continuity : $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

x Navier–Stokes : $\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$

y Navier–Stokes : $\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$

- In 1904 Prandtl simplified these equations inside a BL using the following
  - Re number is high ($Re \gg 1$) and BL is thin ($\delta \ll L$).
  - Vertical velocities are small ($v \ll u$)
  - $\frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}$ (changes in y direction are much more rapid than those in x direction)

Prandtl's BL Equations (cont'd)

- An "order of magnitude analysis" using Prandtl's simplifications gives (see the handout)

Continuity : $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

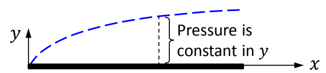
x N–S : $\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$ (small: $\frac{\partial^2 u}{\partial x^2}$)

y N–S : $\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$ (small: $u\frac{\partial v}{\partial x}$, small: $v\frac{\partial v}{\partial y}$, small: $\frac{\partial^2 v}{\partial x^2}$, small: $\frac{\partial^2 v}{\partial y^2}$)

- IMPORTANT: Third equation says "Inside the BL pressure does not change in the y direction".
- IMPORTANT: Inside the BL, $p=p(x)$ only. Pressure inside the BL is treated to be known, which is something imposed by the outer inviscid flow.

Pressure Change Inside a BL

p_{outside} depends on U as given by the Bernoulli (or Euler) equation $\frac{dp}{dx}=-\rho U\frac{dU}{dx}$



- If the freestream velocity (U) outside the BL is constant
  - Euler eqn. for the outside flow tells that outside pressure is also constant.
  - Pressure inside the BL is also constant.
  - Inside the BL there is a balance of inertia and viscous forces.
- If the freestream velocity (U) outside the BL changes in x direction
  - Pressure outside and inside the BL will also change in the x direction.
  - Inside the BL there is a balance of inertia, viscous and pressure forces.

Pressure Change Inside a BL (cont'd)

Flat Plate

U is constant

p_{outside} is constant

p inside the BL is constant

Diverging Duct

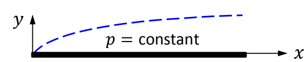
U is decreasing

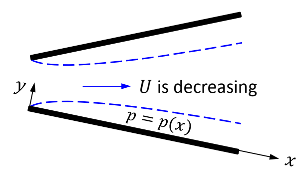
p_{outside} is increasing

p inside the BL is increasing in the x direction

Inside the BL $p=p(x)$

U is constant

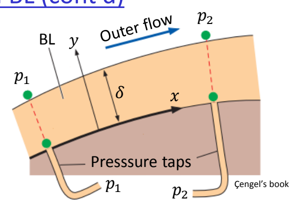


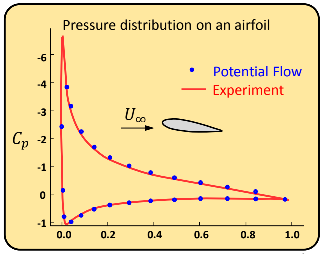


Exercise : How will the pressure inside the BL vary for flow over a cylinder?

Pressure Change Inside a BL (cont'd)

- Static holes are used to measure pressure distribution on a wall.
- Since p does not change in the y direction inside the BL, the measured pressures are equal to the ones at the edge of the BL.
- Therefore an inviscid Euler solution or even a potential flow solution for the flow outside the BL may predict correct pressure distribution over the surface.
- This is the reason why inviscid flow solutions over streamlined bodies such as airfoils can predict the lift force accurately.
- It is not the case for airfoils at high angle of attack or for blunt bodies due to flow separation. BL theory is not valid after the separation point.





Blasius' Exact Solution of BL over a Flat Plate

- In 1908 Blasius, a student of Prandtl, obtained the analytical solution of the following BL equations for laminar flow over a flat plate with zero pressure gradient.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

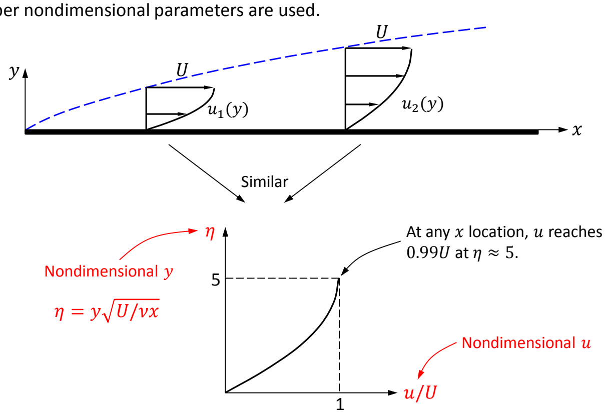
$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2}$$

dp/dx is zero for flat plate

- He used the following boundary conditions
  - At $y = 0$: $u = 0$ & $v = 0$ (No-slip)
  - As $y \to \infty$: $u \to U$ (Asymptotic approach to free stream velocity)
- Blasius' solution is valid for laminar flow over a flat plate with no pressure gradient.
- The solution uses streamfunction and a similarity transformation. You are NOT responsible for its details.

Blasius' Exact Solution (cont'd)

- Blasius cleverly used the fact that velocity profiles at all x sections are similar if proper nondimensional parameters are used.

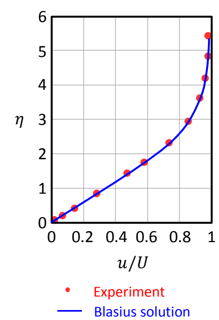


Blasius' Exact Solution (cont'd)

- Velocity profile of Blasius' exact solution has no analytical expression.
- The solution is usually obtained by a numerical solution and the results are given as a table.

η	u/U	η	u/U
0.0	0.00000	2.4	0.72898
0.1	0.03321	2.6	0.77245
0.2	0.06641	2.8	0.81151
0.3	0.09960	3.0	0.84604
0.4	0.13276	3.5	0.91304
0.5	0.16589	4.0	0.95552
0.6	0.19894	4.5	0.97951
0.8	0.26471	5.0	0.99154
1.0	0.32978	5.5	0.99688
1.2	0.39378	6.0	0.99897
1.4	0.45626	6.5	0.99970
1.6	0.51676	7.0	0.99992
1.8	0.57476	8.0	1.00000
2.0	0.62977	9.0	1.00000
2.2	0.68131	10.0	1.00000

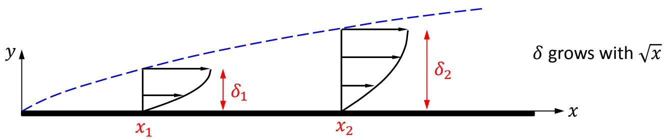
Çengel's book



Blasius' Exact Solution (cont'd)

- u reaches $0.99U$ at $\eta \approx 5$.
- In other words when η becomes 5, y becomes δ.
- Using the definition of η, BL thickness at any x location becomes

$$\eta = y\sqrt{U/\nu x} \quad \rightarrow \quad 5 = \delta\sqrt{U/\nu x} \quad \rightarrow \quad \boxed{\delta = 5\sqrt{\frac{\nu x}{U}}}$$



δ grows with $\sqrt{x}$

- Using local Reynolds number definition, $Re_x = Ux/\nu$

$$\boxed{\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}}$$

Most important result of Blasius' solution

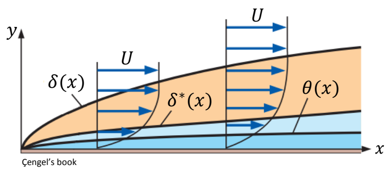
Blasius' Exact Solution (cont'd)

- BL displacement thickness at any section

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \frac{1.721\,x}{\sqrt{Re_x}}$$

- Momentum thickness at any section

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \frac{0.664\,x}{\sqrt{Re_x}}$$



$\delta^* \approx 0.35\,\delta$

$\theta \approx 0.14\,\delta$

$H = \delta^*/\theta \approx 2.6$

Shape factor

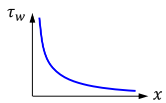
Blasius' Exact Solution (cont'd)

- Shear stress at the wall

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{0.332\,\rho U^2}{\sqrt{Re_x}}$$

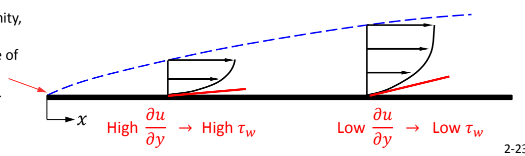
- Local skin friction coefficient (dimensionless wall shear stress)

$$C_{fx} = \frac{\tau_w}{\rho U^2/2} = \frac{0.664}{\sqrt{Re_x}}$$



- As x increases τ_w decreases. $\tau_w \sim 1/\sqrt{x}$

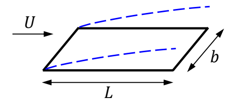
At $x = 0$, τ_w shoots to infinity, which is unphysical. This is because at the leading edge of the plate Prandtl's BL simplifications are not valid.



Blasius' Exact Solution (cont'd)

- Drag force acting on one side of a plate of length L and width b

$$F_D = \int_{x=0}^{L} b\tau_w dx = \frac{0.664\,\rho U^2 bL}{\sqrt{Re_L}}$$



where $Re_L = UL/\nu$.

- Overall skin friction coefficient (dimensionless drag force)

$$C_{fL} = \frac{F_D}{\rho bLU^2/2} = \frac{1.328}{\sqrt{Re_L}}$$

Exercise: A wind tunnel's square test section is 1 m long. Assuming uniform laminar flow at the inlet of the test section determine the maximum allowed air speed such that the BL remains laminar inside the test section. For this air speed determine δ, δ^* and θ at the exit of the test section.

Von Karman's Approximate Momentum Integral Approach (MIA)

Blasius' Exact Solution

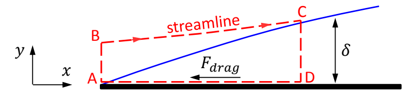
- laminar BL
- over a flat plate
- for zero pressure gradient $\left(\frac{dp}{dx} = 0\right)$

Momentum Integral Approach (MIA)

- both laminar and turbulent BLs
- over flat and curved surfaces
- for any known $U(x)$ and the corresponding $p_{\text{outside}}(x)$
- but needs a velocity profile guess and provides approximate results

MIA for a Flat Plate

- Consider 2D, steady, incompressible flow over a flat plate.
- MIA is based on the mass and momentum conservation for a differential control volume inside the BL.



- Continuity equation : $\dot{m}_{AB} + \dot{m}_{CD} = 0$
- x–momentum equation : $\sum F_x = \text{Mom. flux}_{AB} + \text{Mom. flux}_{CD}$

For the flat plate, pressure is constant everywhere and the only external force is the drag force.

MIA for a Flat Plate (cont'd)

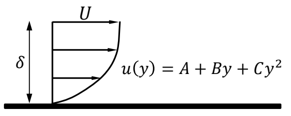
- Following the derivation given in the handout, we get

$$\tau_w = \rho U^2 \frac{d\theta}{dx}$$

- This is von Karman's momentum integral equation.
- This version is valid for flow over a flat plate, i.e. U and p are constant, not changing with x. It is valid for both laminar and turbulent BLs.
- To work with this equation we need to start with an assumed velocity profile inside the BL.
- Then we can calculate all BL parameters such as $\delta(x)$, $\theta(x)$, $\tau_w(x)$, etc.
- The power of the approach is that it gives acceptable results even when the assumed velocity profile is rather crude.

Using MIA for Laminar BL over a Flat Plate

- Step 1. Assume a suitable velocity profile for $u(y)$ inside the BL
- For laminar BLs polynomials of different degrees or trigonometric functions can be used.
- For example a quadratic polynomial profile inside a BL is



Determine the constants A, B and C using three boundary conditions

- $u(0) = 0$
- $u(\delta) = U$
- $\left.\frac{\partial u}{\partial y}\right|_{y=\delta} = 0$

$$A = 0, \quad B = \frac{2U}{\delta}, \quad C = -\frac{U}{\delta^2}$$

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Using MIA for Laminar BL over a Flat Plate (cont'd)

- Step 2. Express τ_w in terms of the assumed velocity profile.
- For laminar flows we can use $\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$.
- For the parabolic profile of the previous page $\tau_w = \mu \frac{2U}{\delta}$.
- Step 3. Use this τ_w in the momentum integral equation and determine $\delta(x)$.
- After finding $\delta(x)$ other details inside the BL can be computed.
- Following table provides a summary of results of MIA for various velocity profiles.

	$\delta\sqrt{Re_x}/x$	$C_{fx}\sqrt{Re_x}$	$C_{fL}\sqrt{Re_L}$
MIA, Linear profile	3.46	0.578	1.156
MIA, Parabolic profile	5.48	0.730	1.460
MIA, Cubic profile	4.64	0.646	1.292
MIA, Sine profile	4.79	0.655	1.310
Blasius (exact)	5.00	0.664	1.328

Using MIA for Laminar BL over a Flat Plate (cont'd)

Exercise : Use the MIA with linear velocity profile and obtain the numbers of the 1st row of the previous table.

Exercise : Air flows at $U = 3$ m/s past a flat plate with a length of $L = 2$ m in the flow direction. Assuming zero pressure gradient and a laminar BL with

a) linear velocity profile
b) parabolic velocity profile
c) cubic velocity profile
d) sinusoidal velocity profile

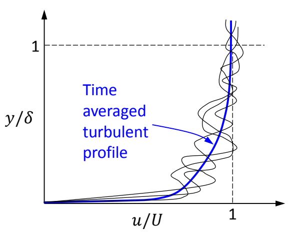
determine δ at $x = L$. Also calculate the drag force acting on the plate. Compare the results with Blasius' solution. Use the values of previous slide's table.

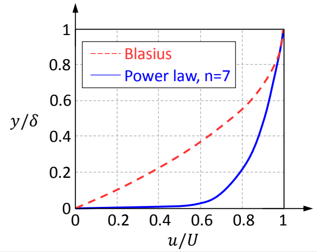
Using MIA for a Turbulent BL

- For turbulent flows power law velocity profile is more appropriate

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/n} \quad \text{where} \quad n = \begin{cases} 7 & 5\times10^5 < Re_x < 10^7 \\ 8 & 10^7 < Re_x < 10^8 \\ 9 & 10^8 < Re_x \end{cases}$$

- Power law profile represents time averaged profile of an unsteady turbulent flow.
- Compared to the Blasius' profile, power law profiles are fuller, with higher speed flow close to the wall.





Using MIA for a Turbulent BL (cont'd)

- Power law velocity profile approximates the time averaged u velocity nicely.
- But it gives infinite slope at the wall, therefore it can not be used to calculate τ_w.
- Instead, experimentally obtained expressions are used, such as

$$\tau_w = 0.0225\rho U^2 \left(\frac{\nu}{U\delta}\right)^{1/4}$$

Exercise: Using power law profile with $n = 7$ and the above τ_w relation in the MIA, obtain the following relations

$$\frac{\delta}{x} = \frac{0.37}{Re_x^{1/5}} \qquad C_{fL} = \frac{0.072}{Re_L^{1/5}}$$

- Note the difference between laminar and turbulent δ's.

Laminar : $\delta \sim x^{1/2}$
Turbulent : $\delta \sim x^{4/5}$
A turbulent BL grows faster than a laminar one.

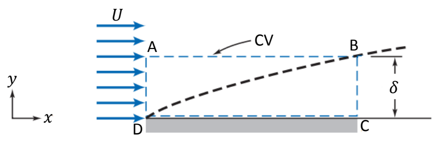
Drag Force Calculation using Integral Formulation (similar to MIA)

Exercise: (Fox's book) A developing boundary layer of standard air on a flat plate is shown below. The free stream flow outside the boundary layer is undisturbed with $U = 50$ m/s. The plate is 3 m wide perpendicular to the diagram. Measurements at section BC revealed that the velocity profile is close to power law profile with $n = 7$ and boundary layer thickness is $\delta = 19$ mm. Calculate

a) the mass flow rates across surfaces AD, AB and BC.

b) the x-momentum flow rates across surfaces AD, AB and BC.

c) the drag force exerted on the flat plate between D and C.



Pressure Gradient Inside a Boundary Layer

$\frac{dU}{dx} > 0$ & $\frac{dp}{dx} < 0$

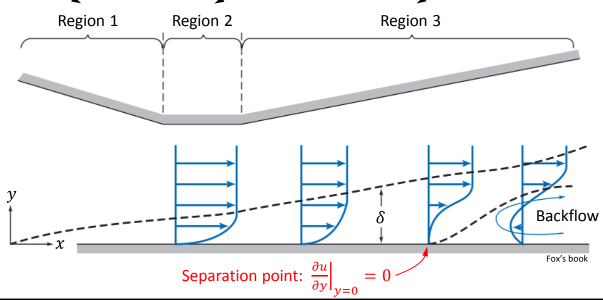
Pressure drops in flow direction (Favorable pressure gradient)

$\frac{dU}{dx} = 0$ & $\frac{dp}{dx} = 0$

Pressure does not change with x (Zero pressure gradient)

$\frac{dU}{dx} < 0$ & $\frac{dp}{dx} > 0$

Pressure increases in flow direction (Adverse pressure gradient)



Pressure Gradient Inside a Boundary Layer (cont'd)

- $\frac{dp}{dx} = 0$: Fluid particles inside the BL slow down due to shear stress only.

  Flow cannot separate from the surface.

- $\frac{dp}{dx} < 0$: Pressure decreases in the flow direction (favorable pressure gradient).

  Pressure force is in the flow direction. It helps the flow attach to the surface even stronger.

  Flow cannot separate from the surface.

- $\frac{dp}{dx} > 0$: Pressure increases in the flow direction (adverse pressure gradient).

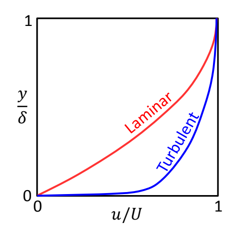
  Pressure force is in the opposite direction of the flow.

  Fluid particles close to the wall with low momentum <u>may</u> come to a stop or even move in the opposite direction of the main flow, called backflow.

- Adverse pressure gradient is the <u>necessary but not sufficient condition</u> for separation. Separation will occur if the adverse pressure gradient is high enough.
- BL theory is no longer applicable after the separation point.

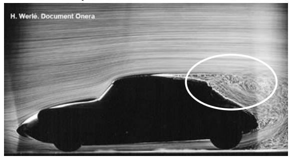
Flow Separation

- Flow separation is generally undesired. It reduces lift force on an airfoil or increases drag force on a blunt body such as a sphere.
- In a diffuser it increases losses and results in poor pressure recovery.
- Turbulent BLs are more resistive to separation because, compared to a laminar one, velocities (and momentum) close to the wall are higher in a turbulent BL.
- Dimples on golf balls and turbulators on wings are used to promote turbulence and delay separation (We'll come to this later).



Flow Separation (cont'd)

Separation over a car

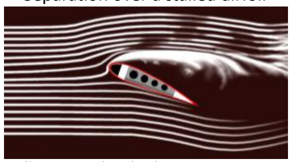


www.onera.fr

A more streamlined concept car

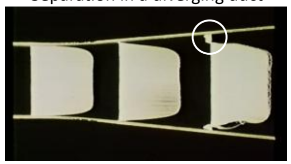
www.germancarforum.com

Separation over a stalled airfoil



http://wings.avkids.com/Tennis/Book/laminar-01.html

Separation in a diverging duct

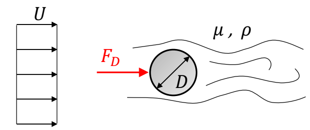


NCFMF films

Drag Force (F_D)

- Component of force acting on a body parallel to the direction of relative motion of the body and the surrounding fluid.
- In ME 305 we studied the dimensionless parameters for the calculation of drag force on a smooth sphere.
- Drag is a function of $F_D = f_1(D, U, \mu, \rho)$
- Buckingham Pi analysis resulted in

$$\frac{F_D}{\rho U^2 D^2} = f_2(Re)$$



- Left hand side is a nondimensional drag force. The result can be expressed by using the drag coefficient C_D

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A} = f(Re)$$

where the area $A = \pi D^2/4$ for the above sphere.

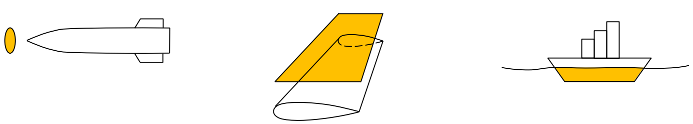
Drag Force (cont'd)

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A}$$

Frontal area

Planform area

Wetted area



- Previous slide says that C_D is a function of Reynolds number only.
- When free surface, compressibility and surface roughness effects are included, C_D becomes a more complicated function of

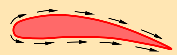
$$C_D = f(Re, Fr, Ma, \varepsilon/L)$$

Drag Force (cont'd)

- First we'll only consider the simple and common case of $C_D = f(Re)$
- F_D is composed of two components.

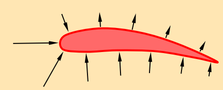
F_D

Friction drag



$$F_{D\,friction} = \int_{body} (\tau_w \, dA)\Big|_{\text{Uniform flow direction}}$$

Pressure drag (Form drag)

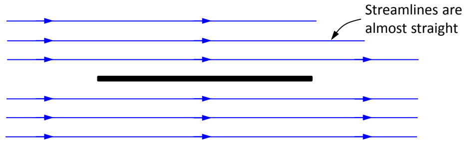


$$F_{D\,pressure} = \int_{body} (p \, dA)\Big|_{\text{Uniform flow direction}}$$

- Obtaining functional forms of τ_w and p distributions for a general flow over even simple shaped bodies is very difficult, if not impossible.
- F_D (or C_D) calculations rely mostly on experimental data.

Drag Force (cont'd)

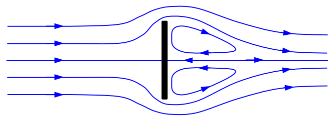
- Flat plate, when aligned parallel to the flow, is a perfectly streamlined body.
- F_D on such a plate is purely do to shear forces. Pressure drag is zero.



$$F_D = F_{D\,friction}$$

$$F_{D\,pressure} = 0$$

- When the plate is perpendicular to the flow the extremely blunt case is obtained.



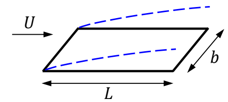
$$F_D = F_{D\,pressure}$$

$$F_{D\,friction} = 0$$

Drag Force on a Flat Plate

Note: Previously we used C_{fL} for flat plate. It's the same as C_D.

- Consider flow over a flat plate, with length L and width b.



$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 bL}$$

Blasius' solution for laminar flow : $C_D = \frac{1.328}{\sqrt{Re_L}}$ (Slide 2–24)

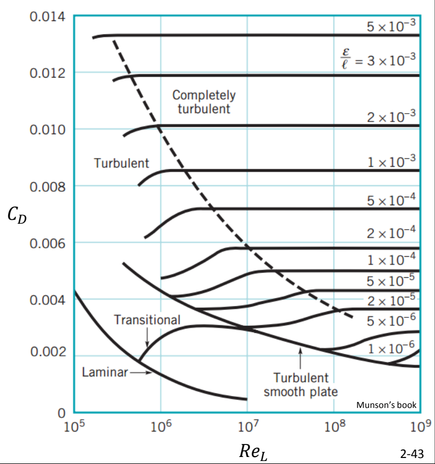
von Karman's MIA with $n = 7$ profile : $C_D = \frac{0.072}{Re_L^{1/5}}$ (Slide 2–32)

Schlichting's formula with transition at 5×10^5 : $C_D = \frac{0.455}{(\log Re_L)^{2.58}} - \frac{1700}{Re_L}$

Completely turb. flow on rough plate : $C_D = \left[1.89 - 1.62 \log\left(\frac{\varepsilon}{L}\right)\right]^{-2.5}$

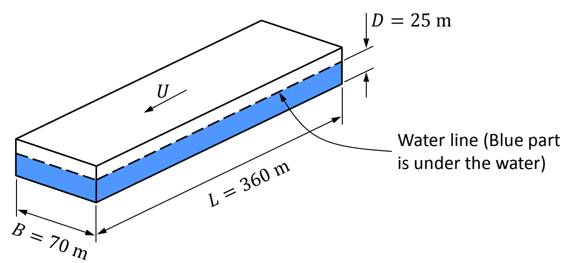
C_D for Flow over a Flat Plate

- Experimental data is plotted.
- Very similar to the Moody diagram used for pipe flow.
- Laminar flow curve represents Blasius' exact solution.
- Relative surface roughness (ε/L) becomes a factor in the turbulent regime.
- For high Re_L values (completely turbulent regime) C_D becomes independent of Re_L, i.e. roughness becomes the only defining parameter.



Drag Force on a Flat Plate (cont'd)

Exercise : A supertanker is 360 m long, has a beam width of 70 m and a draft of 25 m. Estimate the force and power required to overcome skin friction drag at a cruising speed of 13 knot in seawater at 10 °C.



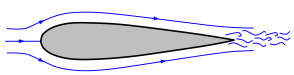
Shape Dependency of Drag

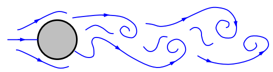
Streamlined bodies

- change the upstream uniform flow minimally
- delay BL separation
- create smaller wake regions
- create less pressure drag

Blunt bodies

- change the upstream uniform flow considerably
- cause early BL separation
- create larger wake regions
- create more pressure drag



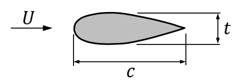


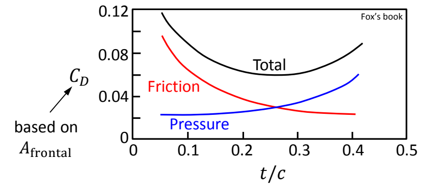
www.germancarforum.com

http://jalopnik.com

Shape Dependency of Drag

- Streamlining the following body by decreasing t/c increases the dominance of friction drag over pressure drag.





- Data is for $Re_c = 4 \times 10^5$.
- Minimum total C_D is 0.06. It is obtained around $t/c = 0.25$.
- This C_D is equal to 20 % of the C_D of a circular cylinder of the same thickness.

Re Dependency of Drag

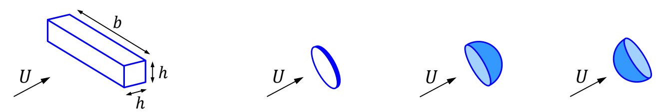
- Re dependency can be studied in three categories.
  - very low Re (inertia forces are negligibly small)
  - moderate Re (laminar BL)
  - very high Re (turbulent BL)

- Flows with $Re < 1$ are called creeping flows.
- These flows are governed by a balance between pressure and viscous forces.
- For creeping flows, irrespective of the shape of the body $C_D \propto \frac{1}{Re}$

$C_D = \frac{20.4}{Re_D}$ $C_D = \frac{13.6}{Re_D}$ $C_D = \frac{24}{Re_D}$ $C_D = \frac{22.2}{Re_D}$

Re Dependency of Drag (cont'd)

- For objects with sharp edges C_D is almost constant for $Re > 1000$.
- Separation always occurs at the sharp edges independent of Re.
- Laminar to turbulent transition has no considerable effect.



$C_D = \begin{cases} 2.05 & b/h = \infty \\ 1.05 & b/h = 1 \end{cases}$ $C_D = 1.17$ $C_D = 1.42$ $C_D = 0.38$

$C_D = 2.30$ $C_D = 1.20$

Dependency of Drag to Turbulence Transition

Streamlined bodies

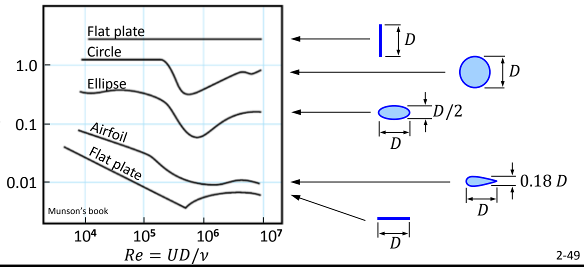
- drag is mostly due to friction
- transition to turbulence increases shear force and increases C_D.

Blunt bodies

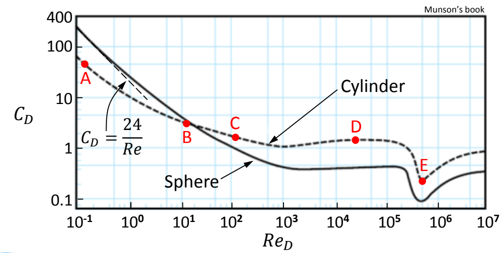
- drag is mostly due to pressure
- transition to turbulence delays separation, decreases size of the wake region and decreases C_D.

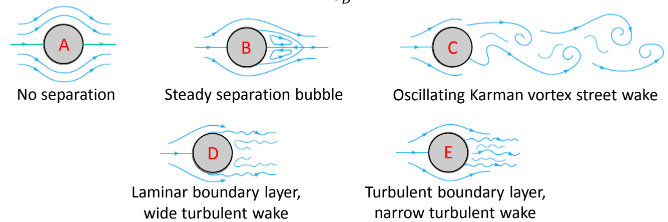
$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 bD}$$

width



Drag Force on a Sphere and Cylinder





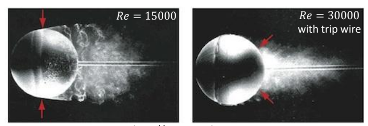
Drag Force on a Sphere (cont'd)

- $Re < 1$: $C_D = 24/Re$, known as Stokes' theory.
- $Re < 1000$: C_D drops with Re.
  At $Re = 10^3$ about 95 % of the drag is due to pressure.
- $10^3 < Re < 4 \times 10^5$: C_D remains almost constant.
- $Re > 4 \times 10^5$: Boundary layer becomes turbulent.
  C_D drops sharply.
- Although C_D seems to be dropping in the entire Re range, F_D actually increases with Re.
- Only at the sudden drop of C_D due to transition, drag force also drops, known as the drag crisis.

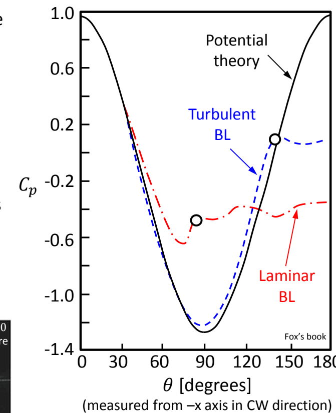
Exercise : Using the figure of the previous slide (or a higher quality one from another source), calculate the drag force on a golf ball sized smooth sphere of 45 mm diameter, flying in a stream of air at 20 °C at different Reynolds numbers and generate a "F_D vs. Re_D" figure.

Drag Force on a Sphere (cont'd)

- Potential flow theory predicts equal pressure on front and back sides.
- For laminar BL pressure on the front side is similar to that of potential theory. However, separation (shown with a circle) occurs at about $\theta = 90°$ and back pressure remains lower than front pressure.
- For turbulent BL pressure on the front side is almost the same as that of potential theory. Separation is delayed up to $\theta = 140°$ and wake is narrower. Back pressure is recovered better.

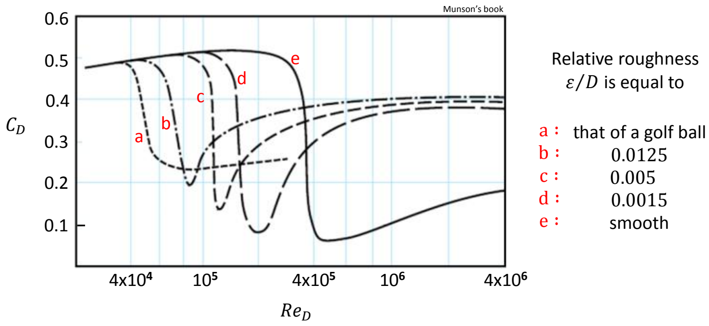


http://sport.maths.org



Drag Force on a Sphere (cont'd)

- For a smooth sphere transition of BL flow into turbulence occurs at about 4×10^5.
- For a rough sphere transition occurs at lower Re values.



Drag Force on a Sphere – Effect of Surface Roughness (cont'd)

- Dimples on a golf ball are carefully designed such that they "trip" the BL flow into turbulence at $Re = 4 \times 10^4$, which is precisely the Re value of a well-hit golf ball.
- A smooth golf ball can at most be hit to approximately 120 m. A dimpled one can fly up to 265 m.

- For a smooth sphere, transition to turbulence reduces C_D by a factor of 5.
- For a golf ball, transition to turbulence reduces C_D by a factor of 2. But the important point is that the reduction takes place at a lower Re, matching the Re of a flying golf ball.

Exercise : Watch the Mythbusters episode about "Fuel Efficiency of a Dimpled Car" and read about a "CFD Study on Golf Ball Aerodynamics".

http://dsc.discovery.com/videos/mythbusters-dimpled-car-minimyth.html

http://www.newswise.com/articles/view/546607

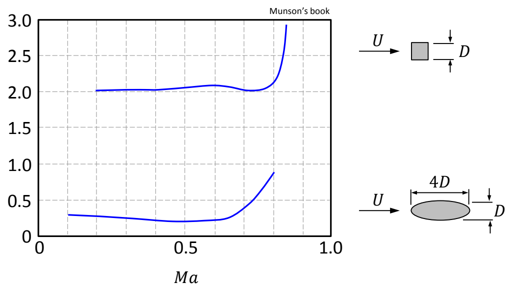
Effect of Mach Number on C_D

- For subsonic with low Mach number, C_D is not a function of Ma.
- But it increases sharply after a certain Ma value.
- Following figure shows the variation of C_D with Ma for two objects of width b. First object has a square cross section.

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 bD}$$

reference density

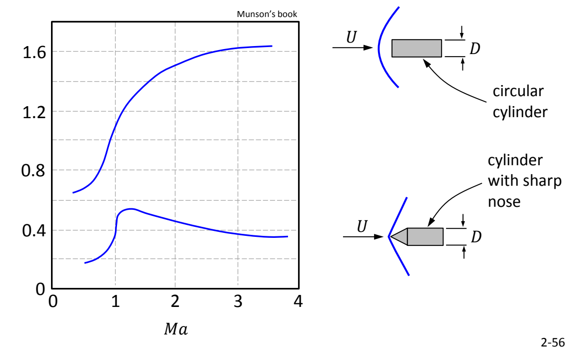
width



Effect of Mach Number on C_D (cont'd)

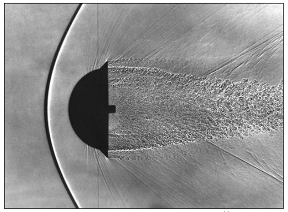
- For supersonic flows formation of shock waves plays an important role.
- Detached, bow shocks form in front of blunt bodies.
- Attached shocks form on the leading edge of pointed bodies.

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 \frac{\pi D^2}{4}}$$

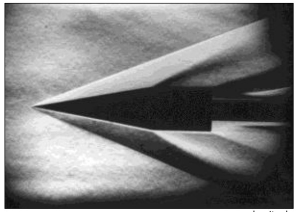


Effect of Mach Number on C_D (cont'd)

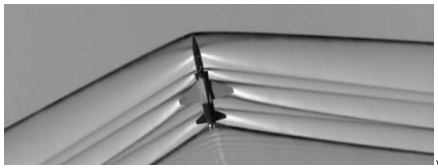
- Schlieren images of shock waves in high speed flight of blunt and pointed bodies.



http://qdl.scs-inc.us



web.mit.edu

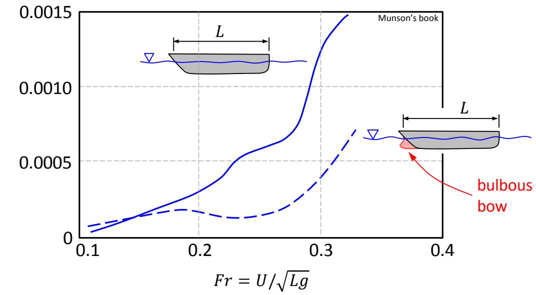


www.nasa.gov

Effect of Froude Number on C_D

- For objects moving on a free surface wave drag becomes important.
- Hull shapes of ships are designed to reduce wave drag.
- Use of a bulbous bow creates extra waves that cancel out the waves generated by the hull and reduce wave drag considerably.

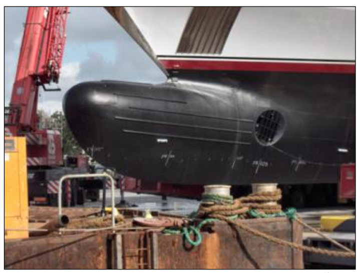
$$C_{D_{wave}} = \frac{F_D}{\frac{1}{2}\rho U^2 L^2}$$



Effect of Froude Number on C_D (cont'd)

- Bulbous bows used for wave drag reduction.

www.globalsecurity.org

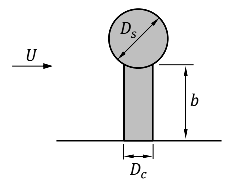


www.cruiseradio.net

Composite Drag

- Approximate drag estimate of a complex body can be done by treating the body as an assembly of simpler parts. See the distributed handout.
- For example for an airplane, drag on the fuselage, wings and tail can be estimated separately and added.
- Interaction between various parts affect the accuracy of the analysis.

Exercise : A 95 km/h wind blows past a water tower. Estimate the wind force acting on it (Reference: Munson).



U = 95 km/h

D_s = 12 m

D_c = 4.5 m

L= 15 m

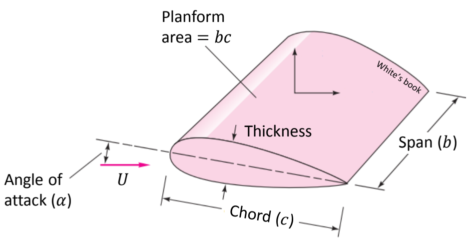
Lift Force

- Lift is the component of the force that is perpendicular to the flow direction.

$$C_L = \frac{F_L}{\frac{1}{2}\rho U^2 A}$$

Lift coefficient

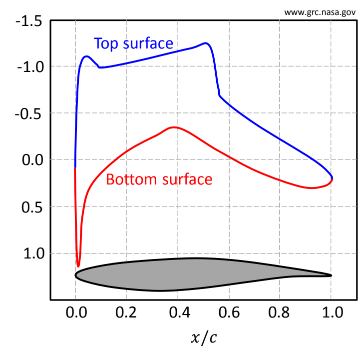
Planform area ($A = bc$)



Lift Force (cont'd)

- An airfoil can generate lift because of the low and high pressures on its top and bottom surfaces, respectively.
- Pressure distribution over an airfoil is usually given as a C_p graph.

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2}$$



Compressible flow with a shock over RAE 2822 airfoil

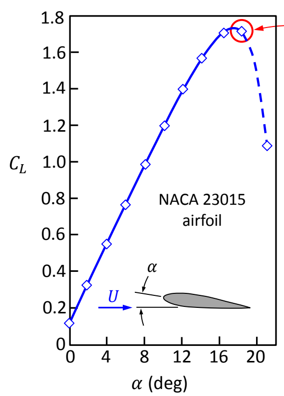
$Ma = 0.729$

$\alpha = 2.31°$

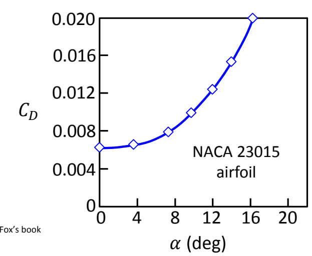
Shock is at the upper surface at about $x/c = 0.55$

Lift Force (cont'd)

- C_L increases with angle of attack up to stall. C_D also increases with angle of attack.



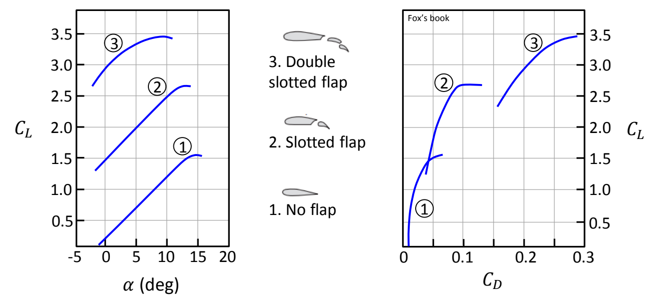
As α increases, separation point on the top surface moves towards the leading edge. At stall, separation occurs at a major portion of the top surface and lift drops sharply.



Fox's book

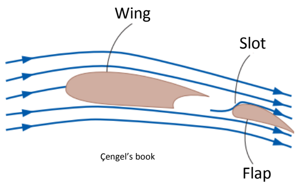
Lift Force (cont'd)

- Flaps are movable portions of a wing trailing edge that may be extended during landing and takeoff to increase effective wing area.
- They increase C_L considerably at a given α. But they also increase C_D and are not used during normal cruising.



Lift Force (cont'd)

- When the flaps are extended air escapes from high pressure region to low pressure region through the slots. This energizes the upper portion of the airfoil and reduces flow separation.
- Extending flaps also increases camber (amount of curvature), which also increases lift.



Lift Force (cont'd)

- Vortex generators can also be used to avoid/delay flow separation.

Lift Force (cont'd)

Exercise: (Çengel's book) A commercial airplane has a total mass of 70,000 kg and a wing planform area of 150 m². The plane has a cruising speed of 558 km/h and a cruising altitude of 12 km, where the air density is 0.312 kg/m³. The plane has double-slotted flaps for use during takeoff and landing, but it cruises with all flaps retracted. Assuming the lift and the drag characteristics of the wings can be approximated by NACA 23012 (given in Slide 2-64), determine

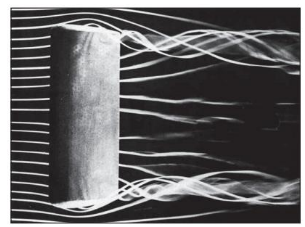
a) the minimum safe speed for takeoff and landing with and without extending the flaps,

b) the angle of attack to cruise steadily at the cruising altitude, and

c) the power that needs to be supplied to provide enough thrust to overcome wing drag.

Induced Drag

- Wings have finite span. At the wing tips high pressure fluid at the bottom escapes to the low pressure upper part, causing trailing edge vortices.
- They increase the drag force known as induced drag.
- They create wake turbulence, which needs to be taken into account by air traffic controllers.



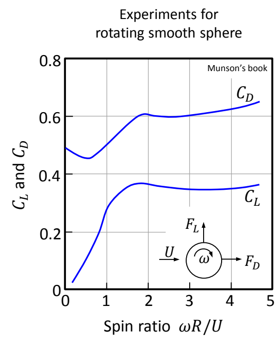
Lift Force (cont'd)

- Birds fly in formation to catch preceding bird's updraft due to wingtip vortices. Read more about it at http://www.sciencemag.org/news/2014/01/why-birds-fly-v-formation

- Winglets (endplates) are used to minimize the undesired effect of wingtip vortices.

Lift on Rotating Bodies (Magnus Effect)

- Lift force is generated on rotating bodies.



Exercise: A table tennis ball weighing 0.0245 N with diameter 3.8 cm is hit at a velocity of 12 m/s with a back spin of angular velocity ω. Determine the value of ω such that lift and drag forces will be balanced and the ball will travel horizontally. Use the data given in the figure. (Reference: Munson)